Cevap Anahtarı

1- a) $(x,y)=t \Rightarrow xa+yb=t,\ a,b \in \mathbb{Z}$

$zt = zxa+zyb \Rightarrow x \mid z \wedge y \mid z'$ ise

$z=xx',\ z=yy' \Rightarrow zt=yxy'a+xx'yb \Rightarrow zt=xy(ya+x'b)$

$xy \mid zt$ bulunur.

b) $\alpha \mid 27+23i \Rightarrow d(\alpha) \mid 1258$ $\quad d(\alpha)=1,2,17,34,74,629,1258$

$27+23i=(1+i)(4-i)(6+i)$ bulunur.

2- $x^3-3x^2+27 \equiv 0 \pmod{5}$ çözelim

$b_1=\overline{1}$ $\quad f(1)=25$ $\quad f'(1)=-3 \not\equiv 0 \pmod 5$ 2 hal

$-3k \equiv \frac{-25}{5} \pmod 5 \Rightarrow -3k \equiv -5 \pmod 5$ $\quad 3k \equiv 0 \pmod 5$

$k=0$ $\quad x=1 \pmod{25}$

$f(1)=25$ $\quad f'(1)=-3 \not\equiv 0$ 2 hal geçerli

$-3k \equiv \frac{-25}{25} \pmod 5$ $\quad 3k \equiv 1 \pmod 5 \Rightarrow k \equiv 2 \pmod 5$

$x=1+2.25 \equiv 51 \pmod{125}$ bulunur.

3- a) $n=p^a.q^b \Rightarrow n^2=p^{2a}q^{2b}$ $\quad f(n^2)=77$ ise

$(2a+1)(2b+1)=77=7.11$ $\quad 2a+1=7$ $\quad 2b+1=11$

$a=3$ $\quad b=5$ $\quad f(n^4)=f(p^{4a}.q^{4b}) \Rightarrow$

$(4a+1)(4b+1)=13.21=273$

b) $\sigma(p^2.q^2)=\frac{p^3-1}{p-1}.\frac{q^3-1}{q-1}=(p^2+p+1)(q^2+q+1)=13.31$

$p=3$, $q=5$ bulunur.

4- a) Taplosunu yapınız.

$7x^7 \equiv 5 \pmod{23} \Rightarrow \text{ind}7 + 7\,\text{ind}x \equiv \text{ind}5 \pmod{22}$

$7\,\text{ind}x \equiv 1 - 19 \equiv 4 \pmod{22} \Rightarrow 7\,\text{ind}x \equiv 4 \pmod{22}$

$21\,\text{ind}x \equiv 12 \pmod{22} \Rightarrow -\text{ind}x \equiv 12 \pmod{22}$

$\text{ind}x \equiv 10 \pmod{22} \Rightarrow x \equiv 5^{10} \pmod{23}$

$\Rightarrow x = 9$ bulunur.

b) $\left(\frac{170}{131}\right) = \left(\frac{2}{131}\right)^1 \left(\frac{5}{131}\right)^1 \left(\frac{17}{131}\right)^1 = ?$ (1)

$\left(\frac{2}{131}\right) = -1$ $\left(\frac{5}{131}\right)^1 \left(\frac{131}{5}\right) = (-1)^{2.65} = 1$

$\left(\frac{131}{5}\right) = \left(\frac{1}{5}\right) = 1$

$\left(\frac{17}{131}\right)^1 \left(\frac{131}{17}\right) = (-1)^{8.65} = 1$ $\left(\frac{131}{17}\right) = \left(\frac{12}{17}\right) = \left(\frac{2^2 \cdot 3}{17}\right)$

$= \left(\frac{3}{17}\right)^1$

$\left(\frac{3}{17}\right)\left(\frac{17}{3}\right) = (-1)^{1.8} = 1$ $\left(\frac{17}{3}\right) = \left(\frac{2}{3}\right) = -1$

5- a) $x = [2, \overline{1,4}]$ $y = [1,4,1,4\ldots]$

$y = 1 + \frac{1}{4 + \frac{1}{y}} = \frac{5y+1}{4y+1} \Rightarrow 4y^2 - 4y - 1 = 0$ $y = \frac{1+\sqrt{2}}{2}$

$x = 2 + \frac{1}{y} = 2 + \frac{2}{1+\sqrt{2}} = 2\sqrt{2}$ bulunur.

b) $\sqrt{7} = [2, \overline{1114}]$ bulunur.

$$\sqrt{7} = 2 + \frac{1}{x_1}, \quad \sqrt{7} - 2 = \frac{1}{x_1}, \quad x_1 = \frac{1}{\sqrt{7}-2} \Rightarrow x_1 = \frac{\sqrt{7}+2}{3} *$$

$$\frac{\sqrt{7}+2}{3} = 1 + \frac{1}{x_2} \Rightarrow x_2 = \frac{3}{\sqrt{7}-1} = \frac{\sqrt{7}+1}{2}$$

$$\frac{\sqrt{7}+1}{2} = 1 + \frac{1}{x_3} \Rightarrow x_3 = \frac{\sqrt{7}+1}{3}$$

$$\frac{\sqrt{7}+1}{3} = 1 + \frac{1}{x_4} \Rightarrow x_4 = \sqrt{7}+2$$

$$\sqrt{7}+2 = 4 + \frac{1}{x_5} \Rightarrow x_5 = \frac{\sqrt{7}+2}{3} *$$

$\sqrt{7} = [2, \overline{1,1,1,4}]$ bulunur.